Bahrain Polytechnic بوليتكنك البحرين

The Mathematics of Getting to the Moon

A Case Study of Problem Based Learning

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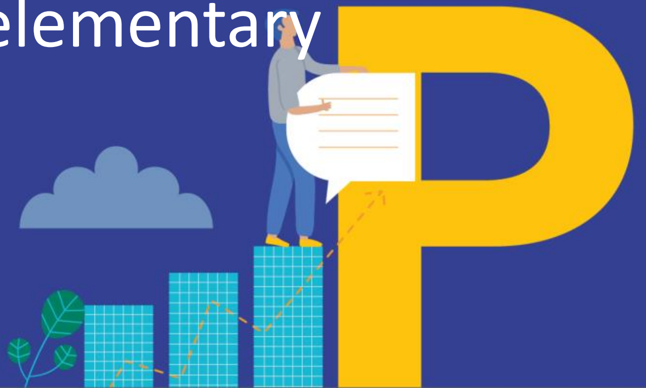
1. Introduction and Statement of the Problem

- DOMAIN: This paper is concerned with trying to shed more light on the problems that are suited to problem-based learning (PBL).
- MOTIVATION: It is important because we want to ease the process of selecting suitable problems.

1. Introduction and Statement of the Problem

- OBJECTIVE: To devise a checklist to assist in determining whether or not a problem is suited to PBL.
- In order to aid understanding of the items on the checklist, reference is to be made to a problem the author considers as well-suited to PBL.
- As an example, the author tackled a problem posed by (Beauzamy, 2016).
- The problem is how to obtain the flight times of a rocket trajectory reaching a specific target. The solution involves using elementary dynamics.



1. Introduction and Statement of the Problem

- HYPOTHESIS: H1: The provision of a checklist makes it easier to determine whether or not a problem is suited to PBL.

2. Background – Related Theory

- PBL theory
- The reference problem concerns the motion of a rocket. The mathematics for this is Ordinary Differential Equations (ODEs).

2. Background – Used and Considered Technologies

- The reference problem was solved using the Octave programming language. The computer program was written by the author.
- No other technology was considered.

2. Background – Literature Review

- PBL – A search was made for PBL articles that refer to the ‘features’ or ‘characteristics’ of a problem.
- ODEs – A search was made for articles that used PBL to teach ODEs.

2. Background – Market Research

- No search was made for software that calculates rocket trajectory times.
- The author is not aware of any such software.

3. Product or Solution?

- The author was interested in solving the rocket trajectory problem.
- Although the computer program that has been written can be used by others, it lacks aspects that make it suitable for release as a product.
- e.g. No attempt has been made to produce a computer program that is easily usable by others.
- With further work, the computer program could be enhanced to make it a product.

4. The Reference Problem

- One must create a model of the scenario. The first step in creating a model is to break the problem down into separate pieces.
- Model: Rocket (mass [e.g. 1 tonne], shape [e.g. sphere of radius 1m], etc.); Earth's atmosphere (e.g. see Table 1); etc.

Table 1: Air density at different altitudes.

Altitude (m)	Density (kg/m^3)
0	1.22500
5000	0.736116
…	…
50000	0.000977525

4. The Reference Problem

- I do not know whether the model I chose is the standard analytical model for solving rocket flight problems.
- Key formula: The formula used to calculate air resistance (drag) is:

$$F_D = \frac{1}{2}\rho V^{\alpha} C_D A$$

- The parameters used in the formula are shown in Table 2.

Table 2: The model's parameters

	Interpretation	Values
F_D	Drag force	
ρ	Density of air	See Table 1
V	Speed of the rocket (m/s)	
α	A coefficient. Its value depends on the object's speed. Our object is travelling at hypersonic velocity.	Between 2 and 2.5
C_D	Drag coefficient	0.5 for a sphere
A	Cross-sectional area of sphere (m^2)	π

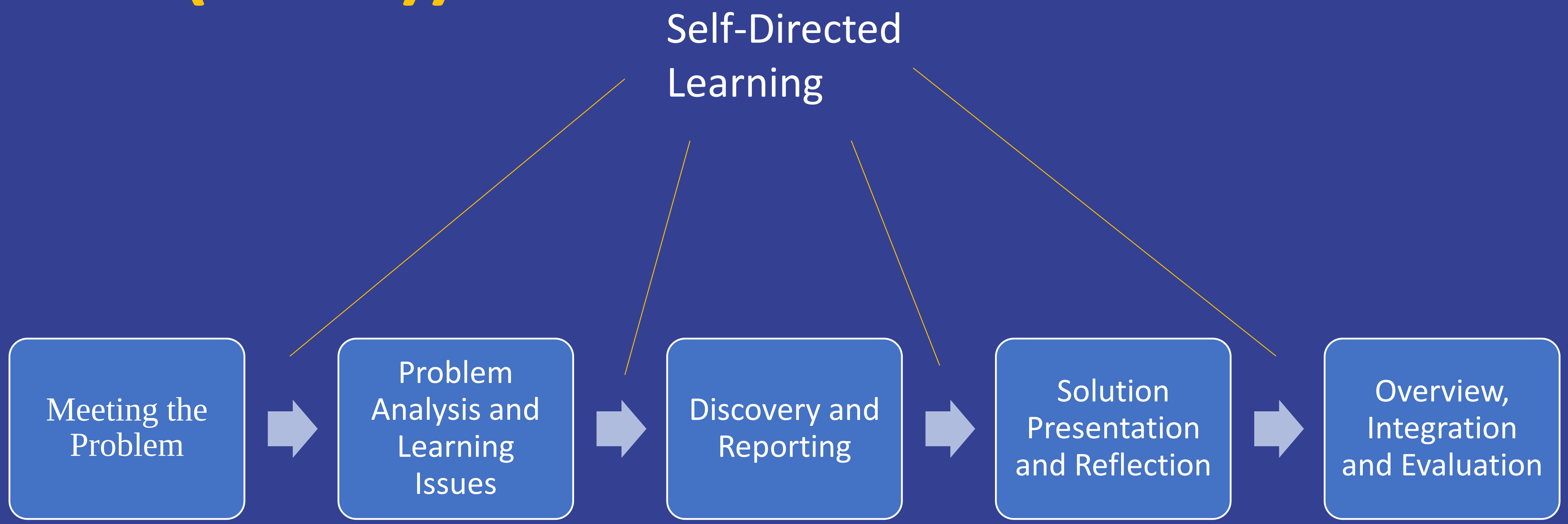
5. Limitations of Study

The study described here considers only one reference problem.

6. Literature Review

- In this research, interest focuses on identifying those problems that are suited to PBL.
- “An effective problem must first engage students’ interest and motivate them to probe for deeper understanding of the concepts being introduced …” (Duch *et al.*, 2001)
- Let us now consider what is meant by the PBL process. Fig. 1 gives a breakdown of the PBL process.

Figure 1: The PBL process (based on Tan (2002)).



6. Literature Review

- The research described here concerns the study of ordinary differential equations (ODEs).
- Lewis & Powell (2016) used PBL in college math classes for teaching ODEs amongst other things.
- Dian & Apriani (2019) used PBL to teach ODEs.

7. Identifying Features of a Problem that are Supportive of PBL

Some features of a problem that are supportive of PBL:

- the subject matter supports the curriculum
- the problem is topical
- it is easy for learners to see how the subject matter is useful in the real world
- students can be asked to identify and discuss any strong assumptions that have been made in the problem
- for STEM disciplines: solving the problem requires students to use ICT, if not be engaged in some computer programming

7. Identifying Features of a Problem that are Supportive of PBL

- the problem can be broken down into 'chunks.' Students can be asked to tackle one chunk to introduce the problem.
- students can tackle one, many, or all chunks. This means that the problem can be an individual assessment or a group assessment.
- the extent of the problem is such that it can be given as a final year project.
- after tackling a chunk, students should have a rough idea of whether their solution is sensible or not, with reference to the real world.
- for STEM disciplines: the problem should be suited to being solved using a 4th generation computer language, such as Matlab or Octave, rather than having to resort to the intricacies of a language such as Java (unless, of course, the students are computing undergraduates).

7. Identifying Features of a Problem that are Supportive of PBL

- the subject matter should have a degree of uncertainty, as large real-world problems are not deterministic. There are two aspects to uncertainty:
  - Sensitivity analysis
  - Calculating the probability of an event

7.1 The subject matter supports the curriculum

The example problem is suited to three types of courses:

- a course where ODEs are studied;
- a course where the study of ODEs is a prerequisite and where the curriculum involves their solution on the computer;
- a course where the study of ODEs is a prerequisite and where the curriculum involves the study of the topic of uncertainty.

7.2 Strong assumptions that have been made in the problem

- It is very well-known that rockets are designed with minimum weight as a key constraint. For the current problem, the fact that mass is assumed to be constant is a strong assumption and deserves some discussion. Another point on which it is worth elaborating is the fact that the rocket is assumed to be spherical.
- An example discussion of how the first point above might proceed is now given.

7.2 Strong assumptions that have been made in the problem

- Consider the following formula:

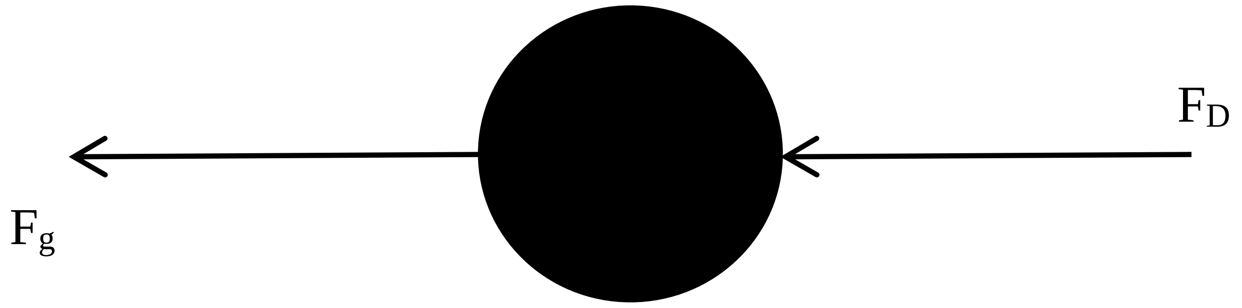
$$\text{Mass ratio} = \frac{\text{Vehicle mass} + \text{propellant mass}}{\text{Vehicle mass}}$$

- For a Boeing 747 the mass ratio is 2, for the X-15 rocket-powered aircraft it was 2.3, for the V-2 it was 3.85.

7.3 The problem can be broken down into ‘chunks’

- The author has identified three stages to solving the problem. These can be broken down into seven ‘chunks.’ An example ‘chunk’: For very large values of α, the thrust runs out before the rocket reaches 50,000 m.
- One or more of these chunks can be given to students. Consider the chunk given above. Students could be asked to first draw a diagram to show the forces acting on the rocket when the thrust has run out. They should come up with something like Figure 2.

Figure 2: Forces acting on the rocket when α is large and the thrust has run out.



7.3 The problem can be broken down into 'chunks'

- Next, students could be asked to derive the equations of motion.
- The sum of the forces acting on the rocket is:

$$\sum F = -mg - DV^{\alpha} = m\frac{dV}{dt}$$

$$\frac{dV}{dt} = -g - \frac{D}{m}V^{\alpha}$$

$$\frac{dy}{dt} = V$$

7.3 The problem can be broken down into 'chunks'

- The second type of exercise that could be given to students is to numerically solve the ODEs that they have found.

8. Conclusion

- An attempt has been made to identify the features of a problem that are supportive of PBL.
- Octave is introduced to students as it is a suitable environment for mathematics-based problems.

Thank you

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